University of Bahrain

College of Information Technology
Department of Computer Science

ITCS253 Discrete Structures II

Second Semester 2014/2015

Exam #2 – 75 Minutes

STUDENT NAME	
STUDENT#	
SECTION	
SERIAL	

This exam contains **5 pages** (including this cover page) and **5 questions**. Check to see if any pages are missing. Enter all requested information on the top of this page.

You are allowed to use Calculators.

You *are not allowed* to use books, notes, or mobiles

Question	Points	Score
1	7	5
2	7	2.5
3	7	7
4	7	6
5	7	4
Total:	35	24.5

Instructor: Dr. Ali Alsaffar Sections# 1 & 2

Answer all questions

(1) Answer the following questions.

(a) **[1 point]** Let $a_n - 2\sqrt{n}a_{n-1} = 3$. Is the relation linear? Why?

1

Yes, because all term ~~...~~ a_n are appd ~~is~~ itself only and doesn't multiplay with another a_n terms.

(b) **[2 points]** Suppose a recurrence relation a_n has the characteristic equation $(x-3)(x-7) = 0$. What is a_n.

$x = 3$, $x = 7$

1

$\therefore a_n = C_1 \cdot 3^n + C_2 \cdot 7^n$

$a_n = 3a_{n-1} + 7^n$

(c) **[2 points]** When we say a graph G has an Euler cycle?

1

We say the graph G has an euler cycle when all ~~vex~~ vertices of it has even degree.

and connected

(d) **[2 points]** Name two invariants of isomorphism.

1 - has n vertices ~~and~~ (that mean have same number of vertices)

2 . Is connected

2

(2) Answer all of the following questions.

(a) **[2 points]** A full *6-ary* tree with 19 vertices. Find the number of internal vertices i and the number of leaves l.

Number of internal vertices :

2

$n = mi + 1 \Rightarrow i = \frac{n-1}{m} = \frac{19-1}{6} = \frac{18}{6} = 3$

Number of leaves

$n = l + i \Rightarrow l = n - i = 19 - 3 = 16$

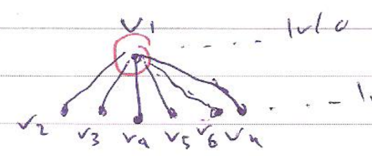
~~$n = \frac{\sum_{i=1}^{n} \deg(v_i)}{2} + 1$~~

(b) **[2 points]** Use the degree equation to find the number of vertices n of a tree if all of its vertices are of degree 1 (leaves). Draw the tree.

0

~~$\sum_{i=1}^{n} \deg(v_i) = 2(n-1)$~~

Graph of tree



because the ~~tree is~~ all vertices of tree is deg (1) (leaves) except the internal vertices ~~there for~~ therefore ~~...~~ we know the heigh is 2 and the graph is full because there is one internal vertices and also it's complete so number of vertices

because all leaves in same level

(c) **[3 points]** Suppose a graph G is connected with n vertices and all vertices are of degree 2. Show that G is not a tree. **Hint:** Use proof by contradiction by assuming that G is a tree.

0.5

Assume G is tree and have 3 vertices one of them of deg 2 and the remaning of degree 1. And assume we put edges between those vertices of degree one then then graph will contain cycle and tree doesn't have cycle therefore the graph will be not tree (Contradiction)

why

(leaves)

(3) **[7 points]** Use Homogeneous technique to solve the following recurrence relation.

$$a_0 = 1, \text{ and } a_n = 5a_{n-1} + 4, \; n \geq 1$$

The charteristic equation:

$(x-5)(x-1) = 0$

The roots:

$r_1 = 5$, $r_2 = 1$

$\therefore a_n = C_1 \cdot 5^n + C_2 \cdot 1^n$

7

The initial value:

$a_1 = 5a_0 + 4 = 5 \times 1 + 4 = 9$

The boundry condition:

$a_0 = 1 \Rightarrow C_1 + C_2 = 1 \; \ldots (1)$

$a_1 = 9 \Rightarrow 5C_1 + C_2 = 9 \; \ldots (2)$

~~by multiplay (1) by~~

by subtract (1) from (2):

$C_1 + C_2 = 1$

$5C_1 + C_2 = 9$

$-4C_1 + 0 = -8$

$-4C_1 = -8 \Rightarrow \boxed{C_1 = \frac{-8}{-4} = 2}$

$\therefore C_1 + C_2 = 1 \Rightarrow 2 + C_2 = 1$

$\Rightarrow \boxed{C_2 = 1 - 2 = -1}$

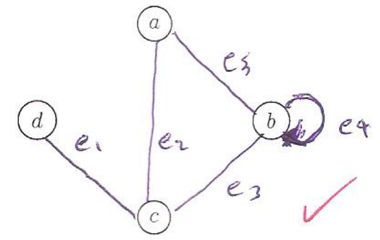
$\therefore a_n = 2 \cdot 5^n - 1^n$

(4) Suppose M is the adjacency matrix for an *undirected* graph G.

$$M = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(a) **[2 points]** Draw the graph using the below vertex ordering.

2



Undirected Graph

(b) **[2 points]** Find the incident matrix of the graph.

2

$$G = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(c) **[1 point]** Is the graph a Bipartite graph? Why?

1

No, because verrex b adjecency itsele

loop.

(d) **[1 point]** Which edges you can remove from G to form a spanning tree.

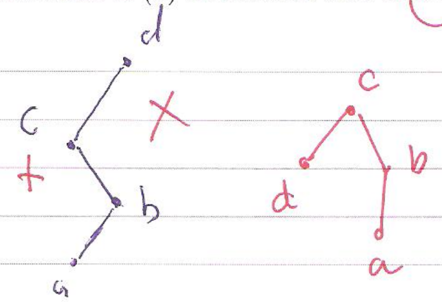
1

~~edges are $\{e_2, e_4\}$~~

edges are e_2 and e_4

(e) **[1 point]** Draw the spanning tree found in (d) as a rooted tree with c as the root.

0



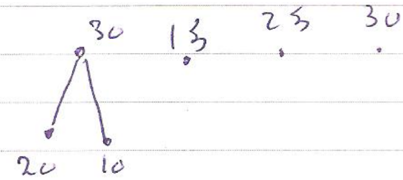
(5) (a) **[5 points]** Construct an optimal Huffman code using the table below.

Character	a	b	c	d	e
Frequency	20	10	15	25	30

(1)

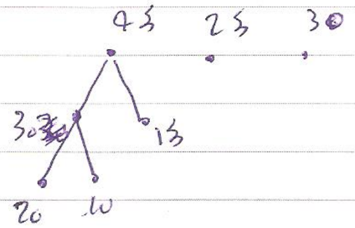
20 10 15 25 30 ← sorted ?

by sum first two:

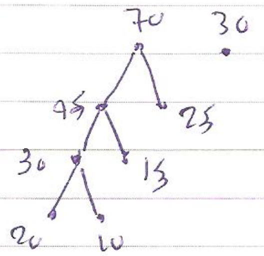


by sum first two:

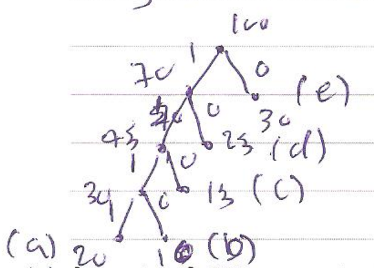
3.5



by sum first two:



by sum last two vertices



(b) **[2 points]** What are the bit codes obtained in part (a) for a, b, c, d, and e, respectively.

a	b	c	d	e
1010101	10101010	101010	1010	~~10~~ 10
1	0			10

0.5